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Mathematics Holiday Homework

1. Show that the relation R on the set R of all real numbers, defined as R = {(a, b): $a \le b^3$} is neither reflexive, nor symmetric nor transitive.
2. Check whether the relation R on R defined by R = {(a, b): $a \le b^3$} is reflexive, symmetric or transitive.
3. Show that the relation R on the set A = {1, 2, 3, 4, 5}, given by R = {(a, b) : | a – b | is even}, is an equivalence relation.Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But, no element of {1, 3, 5} is related to any element of {2, 4}.
4. Prove that the relation R on the set N × N defined by (a, b)R(c, d) ⇔ a + d = b + c for all (a, b),(c, d) ∈ N × N is an equivalence relation.
5. Let A = R – {2} and B = R – {1}. If f : A → B is a mapping defined by $f(x) = \frac{x-1}{x-2}$, show that f is bijective.
6. Show that $f : N \to N$ defined by $f(x) = \begin{cases} \frac{n+1}{2}, & \text{if n is odd} \\ \frac{n}{2}, & \text{if n is even} \end{cases}$ is many-one onto function.
7. Let $f: N \cup \{0\} \to N \cup \{0\}$ be defined by $f(n) = \begin{cases} n+1, if\ n\ is\ even \\ n-1, if\ n\ is\ odd \end{cases}$. Show that f is bijective.
8. Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to Range(f)$ is bijective.
9. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is Bijective.
10. Find the Principal values of each of the following:
    - (i) $tan^{-1}(-\sqrt{3})$
    - (ii) $tan^{-1}(\sqrt{3}) - sec^{-1}(-2)$
    - (iii) $tan^{-1}(1) + cos^{-1}\left(-\frac{1}{2}\right) + sin^{-1}\left(-\frac{1}{2}\right)$
    - (iv) $tan^{-1}(\sqrt{3})) - sec^{-1}(-2) + cosec^{-1}\left(\frac{2}{\sqrt{3}}\right)$
    - (v) $cos^{-1}\left(\frac{1}{2}\right) + 2.sin^{-1}\left(\frac{1}{2}\right)$
    - (vi) $cosec\ \{cos^{-1}a + sin^{-1}a\}$
11. Evaluate:
    - (i) $cos^{-1}\left(cos\frac{7\pi}{6}\right)$
    - (ii) $\cos\left\{cot^{-1}\left(\frac{15}{8}\right)\right\}$
    - (iii) $\tan\frac{1}{2}\left\{cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right\}$
    - (iv) $tan^{-1}\left(tan\frac{7\pi}{6}\right)$
    - (v) $sin^{-1}\left(sin\frac{2\pi}{3}\right)$
    - (vi) $cos^{-1}\left(cos\frac{13\pi}{6}\right)$
12. If x, y, z ∈ [-1, 1] such that $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, find the value of $x^{2006} + y^{2007} + z^{2008} - \frac{9}{x^{2006} + y^{2007} + z^{2008}}$
13. Express each of the following in the simplest form :
    - (i) $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$
    - (ii) $\tan^{-1}\left\{\sqrt{\frac{a-x}{a+x}}\right\}, -a < x < a$
14. Prove that:
    - (i) $\cot^{-1}\left\{\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right\} = \frac{x}{2} \in \left(0, \frac{\pi}{4}\right)$
    - (ii) $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x - a\sin x}\right) = \tan^{-1}\left(\frac{a}{b}\right) - x$
    - (iii) $\tan^{-1}\left\{\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right\} = \frac{\pi}{4} - \frac{x}{2}, \text{if } \pi < x < \frac{3\pi}{2}$
    - (iv) $\tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

15. Construct a 3 × 4 matrix whose elements are given by $a_{ij} = \frac{1}{2}|-3i + j|$.

16. Simplify: $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix} + \sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$.

17. Find matrices A and B, if $2A - B = \begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}$ and $2B + A = \begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$.

18. If $A = \begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and $B = \begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$. Show that AB is a zero matrix of θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

19. Let $A = \begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}$ and I is the identity matrix of order 2. Show that $(I + A) = (I - A).\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$.

20. Find the value of x, if $[1 \quad x \quad 1]\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix} = O$.

21. If $A = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, prove that for all $n \in N$, $(aI + bA)^n = a^nI + na^{n-1}bA$, Where I is the identity matrix of order 2.

22. If $F(x) = \begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$, show that F(x) . F(y) = F(x + y).

23. Express $A = \begin{bmatrix}3 & 2 & 5\\ 4 & 1 & 3\\ 0 & 6 & 7\end{bmatrix}$ as sum of two matrices such that one is symmetric and the other is skew-symmetric.

24. Find the value of k for which the points A(1,-1), B(2,k) and C(4,5) are collinear.

25. Find the value of *k* for which the area of Δ*ABC* having vertices A(2, - 6), B(5, 4) and *C*(*k*, 4) is 35 sq units.

26. If A(-2, 0), B(0, 4) and C(0, k) be three points such that area of ΔABC is 4 sq units, find the value of k.

27. If the points A(a, 0), B(0, b) and C(1, 1) are collinear, prove that $\frac{1}{a} + \frac{1}{b} = 1$.

28. Show that the matrix $A = \begin{bmatrix}-8 & 5\\ 2 & 4\end{bmatrix}$ satisfies the equation $x^2 + 4x - 42 = 0$ and hence find A^{-1}.

29. Using matrices, solve the following system of equations : $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$.

30. Given that: $A = \begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B = \begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$, find A.B. Use this to solve the following system of linear equations: $x - y + z = 4$; $x - 2y - 2z = 9$; $2x + y + 3z = 1$.